Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 9: PL syntax (parse trees)

(a) Show the following expressions are wffs of a PL language with suitable atoms by producing parse trees. Which is the main connective of each wff? What is the scope of each connective in (3) and of each disjunction in (4)? List all the subformulas of (5). Use alternative styles of brackets in (5) and (6) to make them more easily readable.

(1) $((\mathsf{P} \land \mathsf{P}) \lor \mathsf{R})$

(2) $(\neg(\mathsf{R} \land \mathsf{S}) \lor \neg\mathsf{Q})$

(3) $\neg\neg((\mathsf{P} \land \mathsf{Q}) \lor (\neg\mathsf{P} \lor \neg\mathsf{Q}))$

(4) $((((\mathsf{P} \lor \mathsf{P}) \land \mathsf{R}) \land \mathsf{Q}) \lor (\neg(\mathsf{R} \land \mathsf{S}) \lor \neg\mathsf{Q}))$

(5) $(\neg(\neg(\mathsf{P} \land \mathsf{Q}) \land \neg(\mathsf{P} \land \mathsf{R})) \lor \neg(\mathsf{P} \land (\mathsf{Q} \lor \mathsf{R})))$

(6) $(\neg((\mathsf{R} \lor \neg\mathsf{Q}) \land \neg\mathsf{S}) \land (\neg(\neg\mathsf{P} \land \mathsf{Q}) \land \mathsf{S}))$

(b) Which of the following expressions are wffs of a PL language with the relevant atoms? Repair the defective expressions by adding/removing the minimum number of brackets needed to do the job. Show the results are now wffs by producing parse trees.

(1) $((\mathsf{P} \lor \mathsf{Q}) \land \neg\mathsf{R}))$

(2) $((\mathsf{P} \lor (\mathsf{Q} \land \neg\mathsf{R}) \lor ((\mathsf{Q} \land \neg\mathsf{R}) \lor \mathsf{P}))$

(3) $\neg(\neg\mathsf{P} \lor (\mathsf{Q} \land (\mathsf{R} \lor \neg\mathsf{P}))$

(4) $(\mathsf{P} \land (\mathsf{Q} \lor \mathsf{R}) \land (\mathsf{Q} \lor \mathsf{R}))$

(5) $(((\mathsf{P} \land (\mathsf{Q} \land \neg\mathsf{R})) \lor \neg\neg\neg(\mathsf{R} \land \mathsf{Q})) \lor (\mathsf{P} \land \mathsf{R}))$

(6) $((\mathsf{P} \land (\mathsf{Q} \land \neg\mathsf{R})) \lor \neg\neg\neg((\mathsf{R} \land \mathsf{Q}) \lor (\mathsf{P} \land \mathsf{R})))$

(7) $(\neg(\mathsf{P} \lor \neg(\mathsf{Q} \land \mathsf{R})) \lor (\mathsf{P} \land \mathsf{Q} \land \mathsf{R})$

(c*) Show that parse trees for wffs are unique, in the following stages.

(1) Suppose that a wff has the form $(\alpha \land \beta)$ or $(\alpha \lor \beta)$. Then show that the relevant occurrence of the connective '$\land$' or '$\lor$' is preceded by exactly one more left-hand bracket than right-hand bracket. And show that any *other* occurrence of a binary connective in the wff is preceded by at least two more left-hand brackets than right-hand brackets.

(2) You know that if a wff starts with a negation, it must have the form $\neg\alpha$. And if it starts with a left bracket, you now have a method of parsing it uniquely as having either the form $(\alpha \land \beta)$ or $(\alpha \lor \beta)$: what method?

(3) Now develop this into a method of disassembling a complex wff stage by stage, building a parse tree as you go. Confirm that there are no choice points in this process, and if we start with a wff, the result is the only possible one, and therefore parse trees are unique.

(4) As a bonus result, show how the same basic method applied to any string of symbols can be used to decide whether it is a wff or not (because either the process will freeze, or will generate a parse tree).