Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 14: Tautologies

(a) Which of the following wffs are tautologies, which are contradictions, and which are neither?

(1) $\neg((\neg\neg\mathsf{Q} \land \neg\neg\mathsf{P}) \lor \neg(\mathsf{P} \land \mathsf{Q}))$

(2) $(\mathsf{P} \lor ((\neg\mathsf{P} \lor \mathsf{Q}) \land \neg\mathsf{Q}))$

(3) $(\{\mathsf{P} \lor \neg(\mathsf{Q} \land \mathsf{R})\} \lor \{(\neg\mathsf{P} \land \mathsf{Q}) \land \mathsf{R}\})$

(4) $(\{\mathsf{P} \lor (\mathsf{Q} \land \neg\mathsf{R})\} \lor \neg\{(\neg\mathsf{P} \lor \mathsf{R}) \lor \mathsf{Q}\})$

(5) $(\{\mathsf{P} \land (\neg\mathsf{Q} \lor \neg\mathsf{R})\} \lor \neg\{(\mathsf{P} \lor \neg\mathsf{R}) \lor \neg\mathsf{Q}\})$

(6) $\neg(\{\neg(\mathsf{P} \land \neg\mathsf{R}) \land \neg(\mathsf{Q} \land \neg\mathsf{S})\} \land \neg\{\neg(\mathsf{P} \lor \mathsf{Q}) \lor (\mathsf{R} \lor \mathsf{S})\})$

(b*) Which of the following claims are true about PL wffs, and why?

(1) The conjunction of a contradiction and any another wff is still a contradiction.

(2) The conjunction of a tautology and any another wff is still a tautology.

(3) The disjunction of two tautologies is a tautology.

(4) All the tautologies in a PL language express the same truth function as each other.

(5) Every contradiction in a PL language has the same truth table as a wff of the form $(\alpha \land \neg\alpha)$.

If a wff is neither a tautology nor a contradiction, it is said to be *contingent*. Which of the following claims are true, and why?

(6) The negation of a contingent wff is contingent.

(7) The disjunction of two contingent wffs is contingent.

(8) The conjunction of two contingent wffs is contingent.

(9) The disjunction of two contingent wffs is never a contradiction.