Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 15: Tautological entailment

(a) Use the truth table test to determine which of the following arguments are tautologically valid:

(1) $(\mathsf{P} \wedge \mathsf{S}), \neg(\mathsf{S} \wedge \neg\mathsf{R}) \therefore (\mathsf{R} \vee \neg\mathsf{P})$

(2) $\mathsf{P}, \neg(\mathsf{P} \wedge \neg\mathsf{Q}), (\neg\mathsf{Q} \vee \mathsf{R}) \therefore \mathsf{R}$

(3) $(\neg\mathsf{P} \vee \neg(\mathsf{Q} \vee \mathsf{R})), (\mathsf{Q} \vee (\mathsf{P} \wedge \mathsf{R})) \therefore (\neg\mathsf{P} \vee \mathsf{Q})$

(4) $(\mathsf{P} \vee \mathsf{Q}), \neg(\mathsf{Q} \wedge \neg\neg\mathsf{R}) \therefore \neg(\mathsf{R} \vee \mathsf{P})$

(5) $\mathsf{P}, (\mathsf{Q} \vee \mathsf{R}) \ \therefore \ ((\mathsf{P} \wedge \mathsf{Q}) \vee (\mathsf{P} \wedge \mathsf{R}))$

(6) $(\mathsf{P} \vee (\neg\mathsf{P} \wedge \mathsf{Q})), (\neg\mathsf{P} \vee \mathsf{R}), \neg(\mathsf{Q} \wedge \mathsf{S}) \therefore \neg(\neg\mathsf{R} \wedge \mathsf{S})$

(b) Assess the following arguments for validity::

(1) Either Jack went up the hill or Jill did. Either Jack didn't go up the hill or the water got spilt. Hence, either Jill went up the hill or the water got spilt.

(2) Either Jack didn't go up the hill or the water got spilt. Why? Because it isn't the case that Jack went up the hill and Jill didn't. Moreover, it isn't the case that Jill went up the hill and the water got spilt.

(3) Either Jill hasn't trained hard or she will win the race. It isn't true that she'll win the race and not be praised. Either Jill has trained hard or she deserves to lose. Hence either Jill will be praised or she deserves to lose.

(4) Veronika and Marek are both violinists. Veronika and Peter are married but aren't both violinists. And likewise, Marek and Jiří aren't both violinists. So neither Jiří nor Peter are violinists.

(5) Jill is a mother. Either Jack and Jill are both parents, or neither are. Either Jack isn't a parent at all, or he is a proud father. Therefore Jack is a proud father.

(6) Either Popper is a logician or Quine is; moreover it isn't the case that either Russell or Sellars are logicians. Why so? Well, for a start, Popper and Russell are logicians. But not both Quine and Sellars are. And finally, this much is ruled out: Sellars not being a logician while Quine is one, and at the same time Russell being a logician too!