Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 17: The absurdity sign

This set of exercises is very short for two reasons. The principled reason is that there isn't much to be said about the absurdity constant in itself. It comes into its own when we turn to look at a formal proof system for PL *arguments. The less principled reason is that there wasn't much room on the page at the end of Chapter 17!*

(1) Show that for any wff α, $(\alpha \land \bot) \approx \bot$ and $(\alpha \land \neg\bot) \approx \alpha$. What are the analogous results for wffs of the form $(\alpha \lor \bot)$ and $(\alpha \lor \neg\bot)$?

(2) Show that any wff including one or more occurrences of $\bot$ is equivalent to a wff without any absurdity signs or to $\bot$ or to $\neg\bot$.

(3) Find a binary connective % such that a language with just that connective is *not* expressively complete, but a language with % plus $\bot$ *is* expressively complete.

Recalling the notation introduced in Exercises 16(c*), where 'Γ' stands in for some wffs – zero, one, or many:

(4*) Show that $\Gamma \vDash \bot$ if and only if the wffs Γ are tautologically inconsistent.

(5*) Show that $\Gamma, \alpha \vDash \bot$ if and only if $\Gamma \vDash \neg\alpha$.