Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 21

(a) Show that the following inferences can be warranted by proofs using the derivation rules introduced over the last two chapters (i.e. the rules for negation, conjunction and disjunction, as listed in this summary):

(1) $(\mathsf{P} \lor (\mathsf{Q} \land \mathsf{R}))\ \therefore\ ((\mathsf{P} \lor \mathsf{Q}) \land (\mathsf{P} \lor \mathsf{R}))$

(2) $(\mathsf{P} \lor \mathsf{Q})\ \therefore\ \neg(\neg\mathsf{P} \land \neg\mathsf{Q})$

(3) $\neg(\mathsf{P} \land \mathsf{Q})\ \therefore\ (\neg\mathsf{P} \lor \neg\mathsf{Q})$

(4) $(\mathsf{P} \lor \mathsf{Q}),\ (\mathsf{P} \lor \mathsf{R})\ \therefore\ (\mathsf{P} \lor (\mathsf{Q} \land \mathsf{R}))$

(5) $(\mathsf{P} \lor \bot),\ (\mathsf{Q} \lor \bot)\ \therefore\ (\mathsf{P} \land \mathsf{Q})$

(6) $\neg(\mathsf{Q} \land \mathsf{P}),\ ((\mathsf{R} \land \mathsf{Q}) \lor (\mathsf{P} \land \mathsf{Q}))\therefore\ \mathsf{R}$

(7) $(\mathsf{P} \lor \neg\mathsf{Q}),\ (\mathsf{R} \lor \neg\mathsf{P}),\ (\neg\neg\mathsf{R} \lor \mathsf{Q})\ \therefore\ \mathsf{R}.$

(8) $(\mathsf{P} \land (\mathsf{Q} \lor \mathsf{R})),\ \neg((\mathsf{P} \land \mathsf{Q}) \land \mathsf{S}),\ (\neg(\mathsf{P} \land \mathsf{S}) \lor \neg\mathsf{R})\ \therefore\ \neg\mathsf{S}$

(b*) Revisit Exercises 20(b*). Let S be the proof system with our rules for conjunction, negation and now disjunction as well.

(1) Do results (1) to (4) from those previous exercises still obtain now we have revised what counts as the proof system S?

Use similar arguments to those outlined in those previous exercises to show:

(2) If the wffs Γ are S-consistent and $(\alpha \lor \beta)$ is one of those wffs, then either Γ, α or Γ, β (or both) are also S-consistent.

(3) If the wffs Γ are S-consistent and $\neg(\alpha \lor \beta)$ is one of those wffs, then $\Gamma, \neg\alpha, \neg\beta$ are also S-consistent.