Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 28: A first QL language

In the language QL_1, the *proper names* are just

m: Socrates,
n: Plato,
o: Aristotle;

and the *predicates* are just

F: ① is a philosopher,
G: ① is a logician,
H: ① is wise,
L: ① loves ②,
M: ① is a pupil of ②,
R: ① prefers ② to ③.

The domain of quantification for QL_1: people, past and present.

Which of the following expressions are wffs of the language QL_1? In each case, give one possible construction history for those expressions which are wffs, and indicate which logical operators (connectives or quantifiers) are in the scope of which other logical operators. What do those expressions which are wffs mean?

(1) $(\mathsf{Lnn} \wedge (\mathsf{Lmn} \to \mathsf{Lnm}))$
(2) $\forall \mathsf{x}(\mathsf{Fx} \to \mathsf{Lxm})$
(3) $(\mathsf{Go} \wedge \neg\exists \mathsf{x}(\mathsf{Gx} \wedge \mathsf{Hx}))$
(4) $\forall \mathsf{x}(\mathsf{Fx} \to \mathsf{Gx} \wedge \mathsf{Hx})$
(5) $(\exists \mathsf{xFx} \vee \exists \mathsf{xGx})$
(6) $\exists \mathsf{x}(\mathsf{Fx} \vee \exists \mathsf{xGx})$
(7) $(\mathsf{Fx} \vee \exists \mathsf{yGy})$
(8) $\exists \mathsf{x}(\mathsf{Fx} \vee \exists \mathsf{yGy})$
(9) $\forall \mathsf{y} \exists \mathsf{x}\, \mathsf{Rxyx}$
(10) $\exists \mathsf{x} \forall \mathsf{y} \exists \mathsf{x}\, \mathsf{Rxyx}$
(11) $(\mathsf{Lmn} \wedge \forall \mathsf{x}(\mathsf{Lmx} \to \mathsf{Lxn}))$
(12) $(\mathsf{Gn} \to \exists \mathsf{z}(\mathsf{Fz} \wedge \mathsf{Lnz}))$
(13) $\forall \mathsf{y}(\mathsf{Gy} \to \exists \mathsf{z}(\mathsf{Fz} \wedge \mathsf{Lyz}))$
(14) $\neg\exists \mathsf{y}(\mathsf{Fy} \wedge \forall \mathsf{xRxoy})$