Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 38: Relations and Leibniz's Law

(a) Check you know what it is for a relation to be transitive, symmetric and reflexive. In addition

(i) A relation R defined over a given domain is Euclidean just if, whenever a has R to b and a has R to c, then b has R to c;
(ii) R is asymmetric if, whenever a has R to b, then b does not have R to a;
(iii) R is irreflexive if no object has R to itself.

Give examples of relations which are neither reflexive nor irreflexive, and neither symmetric nor asymmetric. Which of the following are true?

(1) If R is asymmetric, it is irreflexive.
(2) If R is transitive and irreflexive, it is asymmetric.
(3) If R is transitive and symmetric, it is reflexive
(4) If R is an equivalence relation, it is Euclidean.
(5) If R is asymmetric and Euclidean, it is irreflexive.
(6) If R is Euclidean and reflexive, it is an equivalence relation.

Give QL proofs for examples of the true claims.

Take the domain containing just the five numbers $0, 1, 2, 3, 4$. How many different equivalence relations can be defined over the domain? (For this purpose, count relations as the same if they have the same extensions.)

(b) In §38.4 we stated the following principle:

> (LL′) Suppose that in the sentence $C(n)$, the context C attributes some property to the object referred to by the term n (the same property whichever term n completes the sentence). Then if a and b are co-referential terms and $C(a)$ is true, then $C(b)$ is also true.

This is quite understandable as it is. But give a version which might satisfy a logician who is *very* pernickety about the use of quotation marks.

(c*) Which, if any, of the following arguments involving identity claims are valid? How do they relate to Leibniz's Law?

(1) Tubby is so-called because of his size. Tubby is none other than Dr Jones. Hence, Dr Jones is so-called because of his size.
(2) Few people have heard of Besarionis dze Jughashvili. Jughashvili is in fact Stalin. Therefore few people have heard of Stalin.
(3) George Orwell is a well-known author. George Orwell is Eric Blair. So Eric Blair is a well-known author.
(4) Necessarily, nine is nine. The number of planets is nine. So, necessarily, the number of planets is nine.