Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 41: $\mathsf{QL}^=$ proofs

(a) Use $\mathsf{QL}^=$ derivations to show the following inferences are valid:

(1) Mrs Jones isn't Kate. So Kate isn't Mrs Jones.
(2) No one who isn't Bryn loves Angharad. At least one person loves Angharad. So Bryn loves Angharad.
(3) If Clark Kent isn't Superman, then Clark isn't even himself. Superman can fly. So Clark can fly.
(4) The goods were stolen by someone. Whoever stole the goods knew the safe combination. Only Jack knew the safe combination. Hence Jack stole the goods.
(5) Take two people (perhaps the same): if the first is taller than the second, the second is not taller than the first. Therefore, if Kurt is taller than Gerhard, they are different people.
(6) There is a wise philosopher. There is a philosopher who isn't wise. So there are at least two philosophers.
(7) Anyone who loves Jo is a logician. Why? Because only one person loves Jo. And some logician loves Jo.
(8) For any number, there's a larger one. There is no number which is larger than itself. So for any number, there's a distinct number which is larger than it.
(9) Exactly one person admires Frank. All and only those who admire Frank love him. Hence exactly one person loves Frank.
(10) The present King of France is bald. Bald men are sexy. Hence whoever is a present King of France is sexy.
(11) Someone is a logician. But no one is the only logician. Therefore there at least two logicians.

(b) The following wffs are alternative renderings of a claim of the form *The F is G*:

(R) $\exists \mathsf{x}((\mathsf{Fx} \wedge \forall \mathsf{y}(\mathsf{Fy} \to \mathsf{y} = \mathsf{x})) \wedge \mathsf{Gx})$
(R′) $\exists \mathsf{x} \forall \mathsf{y}((\mathsf{Fy} \leftrightarrow \mathsf{y} = \mathsf{x}) \wedge \mathsf{Gx})$
(R″) $(\{\exists \mathsf{xFx} \wedge \forall \mathsf{x} \forall \mathsf{y}((\mathsf{Fx} \wedge \mathsf{Fy}) \to \mathsf{x} = \mathsf{y})\} \wedge \forall \mathsf{x}(\mathsf{Fx} \to \mathsf{Gx}))$.

We claimed that from each wff we can derive the other two using a $\mathsf{QL}^=$ proof. Give at least three of the six required proofs. Remember, for us an expression of the form $(\alpha \leftrightarrow \beta)$ simply abbreviates the corresponding expression $((\alpha \to \beta) \wedge (\beta \to \alpha))$.

(c) Outline a proof that 'one and two makes three' (i.e. show that if *There is one F* and *There are two Gs* (where *None of the Fs are Gs*), then *There are three things which are F-or-G*).