Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 2: Validity, soundness, etc.

(a) *Which of the following claims are true and which are false? Explain why the true claims hold good, and give counterexamples to the false claims.*

(1) *The premisses and conclusion of an invalid argument must together be inconsistent.*

Not so: consider the argument *Socrates was a philosopher. So Socrates had a snub nose.* That's plainly invalid as an argument. The premiss and the conclusion are consistent with each other (both true, in fact).

(2) *If an argument has false premisses and a true conclusion, then the truth of the conclusion can't really be owed to the premisses: so the argument cannot really be valid.*

False. The argument 'Socrates is a woman; all women are philosophers; hence Socrates is a philosopher' is a valid argument with false premisses and a true conclusion. To be sure, what makes it the case that Socrates is a philosopher in the actual world is not his being a woman together with all women being philosophers! But it is still the case that *if* (big 'if'!) Socrates were a woman and all women philosophers, then Socrates would in that situation have to be a philosopher – and it is that conditional truth that matters for validity.

(3) *Any inference with actually true premisses and a true conclusion is truth-preserving and so valid.*

False again. What is required for validity is an inference's being necessarily truth-preserving. 'Socrates is a man; hence England has a monarch' has a true premiss and true conclusion but is not necessarily truth-preserving. We can imagine a possible world where Socrates is still a man but England has become a republic.

(4) *You can make a valid argument invalid by adding extra premisses.*

False! If there is no way that A can be true and C false (making the argument *A, so C* valid), then there is no way that A can be true along with B as well, and yet C false (therefore the argument *A, B so C* will be valid too).

The added premiss here will be redundant. But this redundancy doesn't wreck the property of truth-preservation. (And what goes for adding a single premiss goes for adding many premisses.)

(5) *You can make a sound argument unsound by adding extra premisses.*

Yes: just add a false premiss and the argument won't be sound any longer!

(6) *You can make an invalid argument valid by adding extra premisses.*

True. Plainly we can do it sometimes: 'Socrates is a woman; hence Socrates is a philosopher' is invalid. 'Socrates is a woman; all women are philosophers; hence Socrates is a philosopher' is valid.

Can you always make an invalid argument valid by adding a premiss? Yes! Just add the conclusion as a new premiss, and the argument will be trivially valid!

(7) *If some propositions are consistent with each other, then adding a further true proposition to them can't make them inconsistent.*

False! The propositions

> Socrates is a woman. No women are philosophers

are consistent. Add the truth that *Socrates is a philosopher*, however, and we and we get the inconsistent triad

> Socrates is a woman. No women are philosophers. Socrates is a philosopher.

Remember: propositions can be consistent with each other but false!

(8) *If some propositions are jointly inconsistent, then whatever propositions we add to them, the resulting propositions will still be jointly inconsistent.*

True! For example, if there is no way of making A, B, C true together, then there can be no way of making A, B, C, D, E true together.

(9) *If some propositions are jointly consistent, then their denials are jointly inconsistent.*

False! *Jack is a philosopher* is consistent with *Jill is a philosopher*; but equally *Jack is not a philosopher* is consistent with *Jill is not a philosopher*.

(10) *If some propositions are jointly inconsistent, then we can pick any one of them, and validly infer that it is false from the remaining propositions as premisses.*

True. Suppose some propositions A, B, C plus P are inconsistent. That means there is no way of making A, B, C true and making P true too. That means that any way of making A, B, C true must make P false. Which is to say that we can validly infer that P is false from A, B, C.

(b*) *Show that*

(1) *If A entails C, and C is equivalent to C', then A entails C'.*

(2) *If A entails C, and A is equivalent to A', then A' entails C.*

(3) *If A and B entail C, and A is equivalent to A', then A' and B entail C.*

We simply apply definitions to show (1) to (3).

By definition, if A entails C, then in any possible situation where A is true, C is true. And by definition, if C and C' are equivalent, in any situation where C is true, C' is true. Hence if A entails C and C and C' are equivalent, in any situation where A is true, C' is true. So by definition, A entails C'. The other cases are similar.

Can we therefore say that 'equivalent propositions behave equivalently in arguments'?

Yes and no! Swapping a proposition for an equivalent one (whether as premiss or conclusion) won't make a difference to the *validity* of an argument. But being valid is only one of the virtues we want a deductive inference to have! There are other relevant virtues (being convincing for a start), and substituting equivalents in a valid argument might not preserve such other virtues.

Suppose, for example, C and C' are equivalent but very unobviously so. Then the inferences *A, if A then C, hence C* and *A, if A then C, hence C'* must both be valid; but the first can be obvious and convincing while the second might be wildly unobvious and so – presented baldly – completely unconvincing!