NATURAL DEDUCTION RULES FOR PROPOSITIONAL LOGIC

DIAGRAMMATIC SUMMARY OF RULES USED IN IFL2

Rules for negation

(Abs)
$$\begin{array}{c}\gamma\\ \vdots\\ \neg\gamma\\ \vdots\\ \bot\end{array}$$

(RAA)
$$\begin{array}{l}\left|\begin{array}{l}\alpha\\ \vdots\\ \bot\end{array}\right.\\ \neg\alpha\end{array}$$

(DN)
$$\begin{array}{c}\neg\neg\alpha\\ \vdots\\ \alpha\end{array}$$

Rules for conjunction

(∧I)
$$\begin{array}{c}\alpha\\ \vdots\\ \beta\\ \vdots\\ (\alpha\wedge\beta)\end{array}$$

(∧E)
$$\begin{array}{cc}(\alpha\wedge\beta) & (\alpha\wedge\beta)\\ \vdots & \vdots\\ \alpha & \beta\end{array}$$

Rules for disjunction

(∨I)
$$\begin{array}{cc}\alpha & \beta\\ \vdots & \vdots\\ (\alpha\vee\beta) & (\alpha\vee\beta)\end{array}$$

(∨E)
$$\begin{array}{l}(\alpha\vee\beta)\\ \vdots\\ \left|\begin{array}{l}\alpha\\ \vdots\\ \gamma\end{array}\right.\\ \left|\begin{array}{l}\beta\\ \vdots\\ \gamma\end{array}\right.\\ \gamma\end{array}$$

Rules for conditional

(MP)
$$\begin{array}{c}\alpha\\ \vdots\\ (\alpha\to\gamma)\\ \vdots\\ \gamma\end{array}$$

(CP)
$$\begin{array}{l}\left|\begin{array}{l}\alpha\\ \vdots\\ \gamma\end{array}\right.\\ (\alpha\to\gamma)\end{array}$$

Further rules (Could be treated as derived rules, but built into the IFL2 proof system)

(Iter)
$$\begin{array}{c}\alpha\\ \vdots\\ \alpha\end{array}$$

(EFQ)
$$\begin{array}{c}\bot\\ \vdots\\ \alpha\end{array}$$