IFL: Logicbite 5

Deductive validity, logical validity

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We come to one of the most interesting issues touched on in the introductory chapters of *IFL* which lead up to the first Interlude – and this is an issue which I didn't at all do justice to in the first edition.

We defined deductive validity as a matter of necessary truth-preservation. As I noted before, the idea is already in Aristotle:

> A deduction is a discourse in which, certain things having been supposed, something different from the things *results of necessity* because these things are so.

Our paradigm early examples were like the classic 'Socrates is a man; all men are mortal; hence Socrates is a mortal'. The validity here has nothing especially to do with Socrates, or with being a man or being mortal. If we abstract from the featured name and predicates to get 'n is F; all F are G; so n is G', we get a schema which reveals what matters for the validity of the Socrates argument – namely, the meaning of 'all' and the structural way the names and predicates in the premisses and conclusion are related. It is natural to say that the argument here is valid in virtue of exemplifying this abstract form.

But here's another example of an argument involving a necessarily truth-preserving inference step: 'French postboxes are yellow; so French postboxes are coloured'. In this case, however, the validity of the inference does depend on the relation between the predicates 'yellow' and 'coloured'; necessarily, what falls under the first falls under the second. It is natural to say that the postbox argument is valid in virtue of something over and above its abstract form, and depends on the meaning of the featured predicates.

John Buridan brings out the same distinction in Book I, Ch. 4 of his *Tractatus de Consequentiis* (c. 1335-45: the translation is by Stephen Read, and I have very slightly changed one of Buridan's examples to eliminate a distraction):

> Next, in this fourth chapter, a division of consequences is presented, namely, into material and formal.
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> A consequence is called formal if it is valid in all terms retaining a similar form. Or if you want to put it explicitly, a formal consequence is one where every proposition similar in form that might be formed would be a good consequence, e.g., "Some A are B, so some B are A."
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> A material consequence, however, is one where not every proposition similar in form would be a good consequence, or, as it is commonly put, which does not hold in all terms retaining the same form; e.g., "A human is running, so an animal is running," because it is not valid with these terms: "A horse walks, so wood walks." It seems to me that no material consequence is evident in inference except by its reduction to a formal one. Now it is reduced to a formal one by the addition of some necessary proposition or propositions whose addition to the given antecedent produces a formal consequence. E.g., if I say "A human is running, so an animal is running," I will establish the consequence by adding that every human is an animal; for if every human is an animal and a human is running, it follows in a formal consequence that an animal is running. For everyone arguing enthymematically endeavors to prove his consequences in this way if they are not formal.

Set aside Buridan's use of 'proposition' to refer to something of the form *C so D* (indeed, many medieval writers tend to run together *C so D* and *if C then D*). Let's ask: why does Buridan not recognize the running argument as having the form *A human is F, so an animal is F*, a form whose other instances would also be good consequences? Why don't we say that the example argument is valid in virtue of having *that* relevant form?

Well, we *could* say that. But evidently, Buridan won't count necessary relations such as those between *human* and *animal* as formal. Rather, he later writes:

> Now, because we have spoken of the form of a proposition and of the division of consequences into formal and material, it is necessary in this seventh chapter to set down what we say pertains to the form of a consequence and of a proposition and what to the matter.
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> I say that when we speak of matter and form, by the matter of a proposition or consequence we mean ... the subject and predicate, setting aside the syncategoremes [words like 'some', 'are', 'all', 'not', 'if', etc.] attached to them by which they are conjoined or denied or distributed or given a certain kind of supposition; we say all the rest pertains to the form.

In fact, Buridan seems to regard e.g. the human-animal relation as grounded in the nature of things. For him an argument is formally valid just when it depends *only* on its construction. It is formally valid when it depends not on the nature of its subject matter but only on the way that names and predicates are arranged in the premisses and conclusion and are combined together with the likes of 'some' and 'are'. So *form* in his preferred sense is something that can be displayed by an abstract schema using just letters (holding the places for contentful names or predicates) connected with suitably general-purpose logical expressions. (Which isn't to say that is obvious what counts as a general-purpose logical expression!)

It is an old idea that logic, as a systematic form of enquiry, is concerned with arguments whose validity doesn't depend on rather specific connections like the the human-animal relation. For example, in *Posterior Analytics*, Aristotle says that logic

> is not concerned with any determined set of things, or with any one kind;

in other words, logic is concerned with formal principles of reasoning that apply to many different topics. And although ancient writers disagreed about which principles *are* formal enough, they seem to have agreed that logic as a general science is concerned with formally valid arguments – because we can't have a methodological general enquiry into the idiosyncrasies of what Buridan would later call material consequences.

So the principles of reasoning of interest to logic are, traditionally, those that range widely enough for a useful general theory about them to be constructed. Some writers, however, say that logical principles are those that range *universally*, applying to any topic whatsoever. Here's Frege in the preface to his *Begriffsschrift*, saying of logic that

> prescinding from the particularity of things, [it] is based solely on the laws on which all knowledge rests.

Note the 'all'. And from a present-day writer, here is a passage from Warren Goldfarb in his *Deductive Logic* which I've quoted before:

> [T]he principles of logic are general: they do not govern reasoning in one specific subject matter or another, but with reasoning as it applies to any and all areas of study.

Do note the jump here: it is one thing to say with Aristotle that the principles of interest to logic are "not concerned with any one kind", and something else to say with Frege that they "apply to any and all areas of study".

Here's an example to think about. Take the *tense* operators

P: It has at some time been the case that ...
F: It will at some time be the case that ...
H: It has always been the case that ...
G: It will always be the case that ...

Then these arguably have some necessary interrelations. For example, for any φ, $P\varphi$ is equivalent to *not-H-not-*φ and φ implies $GP\varphi$ (for the second, reflect that if Etna is now erupting, it will be always be the case at any future time that Etna has then previously erupted). Such principles are of wide enough application, and can be methodically arranged in enough detail, for some logicians to have constructed formal *tense logics*. But since e.g. mathematical reasoning is about timeless truths (or so we usually suppose) and tensed discourse doesn't sensibly apply, the principles of such a tense logic seemingly "do not apply to any and all areas of study". Compare though 'it is necessarily the case that' and the tense operator 'it is always the case that'. It is usually supposed that the first of these operators *is* of central logical concern. But it seems odd to discriminate sharply between the two (especially as the logical theories of the two operators turn out to have clear parallels).

So which line should we take? Are the principles of reasoning which logic is concerned with those that apply widely, to *enough* areas of study? Or is the proper business of logic really only principles of reasoning of *universal* application?

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Now note that even if we take the second line, and say that logic as a science is concerned with principles of reasoning which right apply across the board, it doesn't follow that material consequences in Buridan's sense aren't genuinely valid deductions. It only follows that logic is only concerned with formal validity, more narrowly, rather than with all cases of deductive validity.

But compare Virginia Klenk in her *Understanding Symbolic Logic*:[1]

> The validity of a particular argument (the instance) depends on its form; again, a particular argument will be valid if and only if its form is valid. Since form has nothing to do with subject matter, it follows that what an argument says, its content, is irrelevant to its validity.

So what about Buridan's argument, a human is running, so an animal is running? Since the only form "which has has nothing to do with subject matter" which this argument instantiates is something like *An F is H, so a G is H* which is invalid, Klenk will have to say the argument is invalid. More generally, for her there are no valid material consequences.

Klenk will presumably agree with Buridan's idea that the argument can be treated as enthymatic for an argument which *is* formally valid, i.e. a human is running, all humans are animals, so an animal is running. But whereas Buridan would say that, since the added premiss is necessary, we now have an explanation of the validity of the original, Klenk is committed to denying the validity of the original.

I prefer, though, to keep using '(deductive) validity' in the wide sense defined by Aristotle, and then we can use 'logical validity' for the notion of being valid in virtue of form, in whatever sense of 'form' logicians should be concerned with. I'd say, then, that Buridan's argument is deductively valid in the wide sense but not logically valid. We'll see later whether this involves more than a terminological difference from Klenk and others who would take her line.

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[1] Virginia Klenk notably wrote *Wittgenstein's Philosophy of Mathematics* (1976), one of the first books to take Wittgenstein's deeply puzzling *Remarks on the Foundations of Mathematics* seriously. Her logic text originally published in 1983 is now in its fifth edition and still widely used.

Suppose again that we do agree with Frege, and say that logic is concerned with forms of argument of universal application. OK: we can start by looking at the way we argue with e.g. 'all', 'some', 'no', 'if', 'and', 'or', etc. Fine. But how do we continue that list of all-purpose 'logical constants'; what else falls within the purview of logic? Where do we draw the line? It isn't obvious! Arithmetic involves necessarily truth-preserving inferences of universal application; do arithmetical principles for this reason count as logical principles too?

I can't delve into these demarcation issues here. In *IFL*, we are only going to be concerned with some indisputable candidates for core logical principles in Frege's narrow sense (I mean it is indisputable that they are candidates, not that they are indisputably correct!). We needn't, for our purposes, then, be concerned about where to draw the outer bounds of logic. I say more about this in *IFL*, Chapter 6.