IFL: Logicbite 6

Propositions

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What sorts of things are the constituents of arguments? We've spoken of premisses and conclusions as being *propositions*, the sort of things that can be true or false, whatever they are. And so far in *IFL* we have been using the term in a theory-neutral way, indeed leaving it open what kind of things propositions might be. But surely we should say something about this!

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Here's a seemingly straightforward no-nonsense view: propositions are just sentences. Or rather, since some sentences are interrogative or imperative – hence not up for assessment as true or false – we'd better say: propositions are just declarative sentences.

But even here, we need more elaboration, because we can talk about sentences in two ways, as *types* and as *tokens*. To explain, here's Nicholas J.J. Smith, writing with characteristic clarity in his *Logic: The Laws of Truth* (2012):[1]

> Consider a word, say, "leisure." Write it twice on a slip of paper, like so:
>
> | leisure leisure |
> |---|
>
> How many words are there on the paper? There are two word tokens on the paper, but only one word type is represented thereon, for both tokens are of the same type. A word token is a physical thing: a string of ink marks (a flat sculpture of pigments on the surface of the paper), a blast of sound waves, a string of pencil marks, chalk marks on a blackboard, an arrangement of paint molecules, a pattern of illuminated pixels on a computer screen—and so on, for all the other ways in which words can be physically reproduced, whether in visual, aural, or some other form. A word token has a location in space and time: a size and a duration (i.e., a lifespan: the period from when it comes into existence to when it goes out of existence). It is a physical object embedded in a wider physical context. A word type, in contrast, is an abstract object: it has no location in space or time—no size and no duration. Its instances word tokens—each have a particular length, but the word type itself does not. (Tokens of the word type "leisure" on microfilm are very small; tokens on billboards are very large. The word type itself has no size.) Suppose that a teacher asks her pupils to take their pencils and write a word in their notebooks. She then looks at their notebooks and makes the following remarks:
>
> (1) Alice's word is smudged.
> (2) Bob and Carol wrote the same word.
>
> ... In remark (1) "word" refers to the word token in Alice's book. The teacher is saying that this token is smudged, not that the word type of which it is a token is smudged (which would make no sense). In remark (2) "word" refers to the word type of which Bob and Carol both produced tokens in their books. The teacher is not saying that Bob and Carol collaborated in producing a single word token between them (say by writing one letter each until it was finished); she is saying that the two tokens that they produced are tokens of the one word type. ...
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> Turning from words to sentences, we can make an analogous distinction between sentence types and sentence tokens. Sentence types are abstract objects: they have no size, no location in space or time. Their instances—sentence tokens—do have

[1] Nick Smith is an Australian logician, who as written extensively on vagueness as a logical issue. His admirable logic text is about at the level of *IFL* and can be warmly recommended.

> sizes and locations. They are physical objects, embedded in physical contexts: arrangements of ink, bursts of sound waves, and so on. A sentence type is made up of word types in a certain sequence; its tokens are made up of tokens of those word types, arranged in corresponding order. If I say that the first sentence of Captain Cook's log entry for 5 June 1768 covered one and a half pages of his logbook, I am talking about a sentence token. If I say that the third sentence of his log entry for 8 June is the very same sentence as the second sentence of his log entry for 9 June, I am talking about a sentence type.

OK, with that distinction in play, how should we elaborate the view that propositions are just declarative sentences. Do we mean they are sentence types or sentence tokens?

Let's continue with a little more of Nick Smith's discussion, since he is so clear:

> Consider a sentence type (e.g., "I am hungry"). A speaker can make a claim about the world by uttering this sentence in a particular context. Doing so will involve producing a token of the sentence. We do not wish to identify the proposition expressed—the claim about the world—with either the sentence type or this sentence token, for the reasons discussed below.
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> To begin, consider the following dialogue:
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> > Alan: Lunch is ready. Who's hungry?
> > Bob: I'm hungry.
> > Carol: I'm hungry.
> > Dave: I'm not.
>
> Bob and Carol produce different tokens (one each) of the same sentence type. They thereby make different claims about the world. Bob says that he is hungry; Carol says that she is hungry. What it takes for Bob's claim to be true is that Bob is hungry; what it takes for Carol's claim to be true is that Carol is hungry. So while Bob and Carol both utter the same sentence type ("I'm hungry") and both thereby express propositions (claims about the world), they do not express the same proposition. We can be sure that they express different propositions, because what Bob says could be true while what Carol says is false—if the world were such that Bob was hungry but Carol was not—or vice versa—if the world were such that Carol was hungry but Bob was not. It is a sure sign that we have two distinct propositions—as opposed to the same proposition expressed twice over—if there is a way things could be that would render one of them true and the other false. So one sentence type can be used to express different propositions, depending on the context of utterance. Therefore, we cannot, in general, identify propositions with sentence types.
>
> Can we identify propositions with sentence tokens? That is, if a speaker makes a claim about the world by producing a sentence token in a particular context, can we identify the claim made—the proposition expressed—with that sentence token? We cannot. Suppose that Carol says "Bob is hungry," and Dave also says "Bob is hungry." They produce two different sentence tokens (one each); but (it seems obvious) they make the same claim about the world. Two different sentence tokens, one proposition: so we cannot identify the proposition with both sentence tokens. We could identify it with just one of the tokens—say, Carol's—but this would be arbitrary, and it would also have the strange consequence that the claim Dave makes about the world is a burst of sound waves emanating from Carol. Thus, we cannot happily identify propositions with sentence tokens.

Now, Nick Smith doesn't say that these are final, knock-down, arguments against treating propositions as sentences: he himself leaves open the possibility of a more complicated story that treats propositions as constructions out of sentences in the final analysis. But you can, I hope, begin

to see why many have thought that propositions indeed aren't sentences – they are, rather what sentences express.

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The idea the primary bearers of truth and falsity are not sentences themselves but what sentences express is ancient. The Stoics, for example, distinguished words from that which is said – i.e. from what they called *lekta*. Those *lekta* which are complete messages are *axiomata* (roughly, the meanings of declarative sentences). And it is strictly speaking *axiomata* rather than the sentences which express them which can be true or false, and so are the topic of logic.

Later, there are medieval disputes about the bearers of truth. Peter Abelard for example distinguishes declarative sentences from the *dicta* which they signify. And, according to some commentators, a *dictum* for Abelard is some kind of abstract entity, neither mental nor belonging to the physical world. (Other commentators disagree – but that's the history of philosophy for you.)

Of course, in more recent philosophy, the key proponent of the view that declarative sentences express other entities which are the prime bearers of truth or falsity is Frege. For him, the sense of a declarative sentence is a *Gedanke* – literally, a thought (crucially, not an act of thinking but rather a possible thought-*content*). And *Gedanken*, says Frege, are those things "for which the question of truth arises". In earlier writings, he talked instead of "judgeable contents". And, he held, logic is primarily concerned with judgeable contents or thoughts, and with the laws governing the relations between them. Frege is emphatic that senses in general and *Gedanken* in particular belong to a "third realm" distinct both from the sensible external world and from the internal mental world, a realm of abstract entities which we can intellectually grasp. So such possible thought-contents (as it were) sit there waiting to be grasped in thought! – and indeed, there are possible contents that might not be expressible in this, that, or the other actual language.

Crudely, then, the Fregean picture is that a sentence (in use) denotes a truth-value[2], but it does so by expressing a proposition (*Gedanke*) and it is propositions which are primarily true or false. A later logician who buys wholeheartedly into this picture is the great Alonzo Church in his *Introduction to Mathematical Logic* (1956):[3]

> [We] declare all true [declarative] sentences to denote the truth-value truth, and all false sentences to denote the truth-value falsehood. In alternative phraseology, we shall also speak of a sentence as *having* the truth-value truth (if it is true) or *having* the truth-value falsehood (if it is false).
>
> The sense of a sentence may be described as that which is grasped when one understands the sentence, or as that which two sentences in different languages must have in common in order to be correct translations each of the other. As in the case of names . . . , it is possible to grasp the sense of a sentence without therefore necessarily having knowledge of its denotation (truth-value) . . . . In particular, though the sense is grasped, it may sometimes remain unknown whether the denotation is truth.
>
> Any concept of a truth-value . . . whether or not it is the sense of some actually available sentence in a particular language under consideration, we shall call a *proposition*, translating thus Frege's *Gedanke*.
>
> Therefore a proposition, as we use the term, is an abstract object of the same general category as a class, a number, or a function. It has not the psychological character of William of Ockham's *propositio mentalis* or of the traditional *judgment*: in the words

[2]If we are being careful, we should say that for Frege a truth-value, *true* or *false*, can be the *Bedeutung* of the sentence, leaving it open to further discussion how to construe 'Bedeutung'.

[3]Church was there in the glory days of the 1930s, working with Kurt Gödel, Alan Turing and Stephen Kleene on the foundations of logic and of computability theory. He has both a famous Theorem and a famous Thesis named after him.

> of Frege, explaining his term Gedanke, it is "nicht das subjective Thun des Denkens, sondern dessen objectiven Inhalt, der fähig ist, gemeinsames Eigenthum von Vielen zu sein [not the subjective performance of thinking but its objective content, which is capable of being the common properly of several thinkers.]"
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> Traditional (post-Scholastic) logicians were wont to define a proposition as a judgment expressed in words, thus as a linguistic entity, either a sentence or a sentence taken in association with its meaning. But in nontechnical English the word has long been used rather for the meaning (in our view the sense) of a sentence, and logicians have latterly come to accept this as the technical meaning of "proposition." This ... provides in English a distinction not easily expressed in some other languages, and makes possible a translation of Frege's *Gedanke* which is less misleading than the word " thought.".
>
> According to our usage, every proposition determines ... (or, as we shall also say, has) some truth-value. ... A proposition is then true if it determines or has the truth-value truth, false if it has the truth-value falsehood.

OK. But how can we further elucidate this Fregean notion of a proposition, inhabiting a third realm but somehow graspable by us?

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Here's plain-speaking Benson Mates in his *Elementary Logic* (1965):[4]

> To some ears it sounds odd to say that sentences are true or false, and throughout the history of the subject there have been proposals to talk instead about statements, propositions, thoughts, or judgments. As described by their advocates, however, these latter items appear on sober consideration to share a rather serious drawback, which, to put it in the most severe manner, is this: they do not exist.
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> Even if they did, there are a number of considerations that would justify our operating with sentences anyway. A sentence, at least in its written form, is an object having a shape accessible to sensory perception, or, at worst, it is a set of such objects. Thus
>
> It is raining
>
> and
>
> Es regnet,
>
> though they may indeed be synonymous, are nonetheless a pair of easily distinguishable sentences. And in general we find that as long as we are dealing with sentences many of the properties in which the logician is interested are ascertainable by simple inspection. Only reasonably good eye-sight, as contrasted with metaphysical acuity, is required to decide whether a sentence is simple or complex, affirmative or negative, or whether one sentence contains another as a part. Even so venerable and strange an issue as whether the conclusion of a sound argument is always somehow contained in the premises is not hard to settle if we take it as referring to sentences.
>
> But matters are quite otherwise when we try to answer the same kinds of questions about propositions, statements, thoughts, and judgments. Propositions, we are told, are the senses or meanings of sentences. They are so-called abstract entities and, as such, are said to occupy no space, reflect no light, have no beginning or end, and so forth. At the same time, each proposition is regarded as having a *structure*, upon which its logical properties essentially depend. If we were to study logic from this point of view, therefore, it would be imperative to have a way of finding out in given cases what that structure is. Unfortunately no simple method is ever given. Since

[4]Benson Mates wrote an early and influential book on Stoic logic, and later a fine book on Leibniz. His widely admired *Elementary Logic* is one of the classic logic texts.

> everyone agrees that sentences of wholly different shapes and structures may have the same meaning, the structures of propositions must not be confused with those of their corresponding sentences; yet a perusal of the literature leaves little doubt that just this sort of confusion frequently takes place. Sometimes, to be sure, one is advised not to rely upon sensory perception at all, but instead to look directly upon the proposition by means of the 'mind's eye'. Whoever admits that he is unable to do this leaves himself open to an obvious and uncomplimentary diagnosis traditionally employed in such cases. But, when all the rhetoric has been expended, we are left once again with the problem of how in practice to ascertain the structure of propositions expressed by given sentences.

And Mates continues in the same vein. But is all hope lost for propositions in something like Frege's sense? Actually, despite Mates's scepticism, there are some stories to be told – and Nick Smith considers some of them in Chapter 11 of his book. Still, debates about such things do get murky and tendentious.

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So on the one hand, there seem to be problems in naively identifying propositions with sentences. On the other hand, once we try to progress beyond the negative parts of the Fregean doctrine (propositions are *not* sentences, are *not* mental items, etc.), it is very unclear how to develop a positive account.

But before we disappear down the rabbit hole, searching out modern theories of propositions, we should pause. There's a prior question. What is going to be the relevance of this hunt to the business of *formal* logic (which is going to involve symbol-juggling, not operating on e.g. Fregean propositions whatever they are)? Maybe we can legitimately set aside a whole swathe of philosophical debate about the propositions involved in ordinary-language arguments as being, after all, not centrally relevant to our purposes. Maybe, as Mates urges, we can indeed stick with sentences for the purposes of formal logic. That would be a relief. Can we do so?

I've now given away much of the core content of my own short chapter on propositions! But I hope the long quotations here have been helpful. So now read *IFL* Chapter 7.