Truth-functions

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Here is Quine in his *Methods of Logic* introducing our next topic ('alternation' is his word for disjunction: and he allows unbracketed multiple conjunctions and disjunctions):

> All that is strictly needed for a precise understanding of negation, conjunction, and alternation is stated in these laws:
>
> > '$\overline{p}$' is true if and only if 'p' is false,
> > '$pq\ldots s$' is true if and only if all of 'p', 'q', ..., 's' are true,
> > '$p \vee q \vee \ldots \vee s$' is true if and only if 'p', 'q', ..., 's' are not all false.
>
> Now it is evident from these laws that negation, conjunction, and alternation share the following important property: in order to be able to determine the truth or falsity of a negation, conjunction, or alternation, it is sufficient to know the truth or falsity of the component parts.
>
> It is convenient to speak of truth and falsity as *truth values*; thus the truth value of a statement is said to be truth or falsity according as the statement is true or false. What we have just observed, then, is that the truth value of a negation, conjunction, or alternation is determined by the truth values of its components. This state of affairs is expressed by speaking of negation, conjunction, and alternation as *truth functions*. In general, a compound is called a *truth function* of its components if its truth value is determined in all cases by the truth values of the components. More precisely: a way of forming compound statements from component statements is *truth-functional* if the compounds thus formed always have matching truth values as long as their components have matching truth values.
>
> The property of truth-functionality which is thus enjoyed by negation, conjunction, and alternation may be better appreciated if for contrast we examine a non-truth-functional compound:
>
> > Jones died because he ate fish with ice cream.
>
> Even agreeing that the components 'Jones died' and 'Jones ate fish with ice cream' are true, we may still dispute over the truth value of this compound. The truth value of the compound is not determined simply by the truth values of the component statements, but by these in company with further considerations; and very obscure those further considerations are. On the other hand the truth value of the conjunction:
>
> > Jones ate fish with ice cream and died
>
> or of the alternation:
>
> > Jones ate fish with ice cream or died
>
> or of the negation:
>
> > Jones did not die
>
> admits of no dispute whatever once the truth values of 'Jones ate fish with ice cream' and 'Jones died' are known individually.
>
> The compound 'p because q' is shown not to be a truth function of 'p' and 'q' by the fact that it comes out true when some truths are put for 'p' and 'q' and false when other truths are put for 'p' and 'q'. In the case of '$p \vee q$, 'pq', and '$\overline{p}$', on the other hand, one true component is as good as another and one false component is as bad as another so far as the truth or falsity of the compound is concerned.

Any particular truth function can be adequately described by presenting a schedule showing what truth values the compound will take on for each choice of truth values for the components. Our three basic truth functions themselves, indeed, were summarily so described [above]. Any unfamiliar fourth truth-functional symbol could likewise be introduced and adequately explained simply by saying what truth values on the part of the components are to make the new compound true and what ones are to make it false. A symbol 'excl-or' for the exclusive 'or', e.g., would be fully explained by a stipulation that 'p excl-or q' is to be false when 'p' and 'q' are taken as both true or both false, and true in the remaining two cases ('p' true and 'q' false or vice versa).

So far so good! And I say much the same in *IFL* §12.1.

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One question arising for non-mathematical philosophers is the use of the notion of a *function* here. I say a bit more about functions in general in §12.2.

Evidently, given what we saw in Chapter 10, any PL wff is a truth-functional compound of its atoms: fixing the values of the atoms uniquely fixes the values of the whole. I say more about this, and about how to set out what Quine calls "a schedule showing what truth values the compound will take on for each choice of truth values for the components" – better known as a truth table – in §12.3. That section also says something about how to cut down the tedium of writing out these truth tables.

Finally, I say something about a complication in §12.4 (but don't worry too much about this on a first read: I'll return to the point in the next Logicbite).

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Now read *IFL*, Chapter 12.