IFL: Logicbite 13

'All possible valuations'

Peter Smith

This Logicbite takes up the theme of *IFL* §12.4. Let me explain why the section – or something like it (no doubt I could improve it!) – is needed.

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First, let's take a quick look at another introductory text book, one we haven't met before. In R.L. Simpson's *Essentials of Symbolic Logic*,[1] sentence letters in use are assigned interpretations via translation keys. That's why Simpson can say

> Since every complete truth table covers all possibilities, there is a row on every complete truth table which corresponds to the actual world. For purposes of doing logic, however, there is no need to determine which row that is.

If the propositional letters were just uninterpreted letters being mapped to truth values, then a row on the truth on the truth-table e.g. assigning $\mathsf{P}, \mathsf{Q}, \mathsf{R}$ respectively the values F, T, T (and then calculating the value of a complex wff involving those atoms) tells us *nothing* about how things are in the world. Such an assignment couldn't "correspond to the actual world". The idea here must be that $\mathsf{P}, \mathsf{Q}, \mathsf{R}$ are interpreted, assigned contentful propositions – so then an assignment of truth values to $\mathsf{P}, \mathsf{Q}, \mathsf{R}$ (and the calculated value of a complex wff on that assignment) gives us a story about the world, which may or may not correspond to how things are in the actual world.

Fine. But later, Simpson tells us that

> A truth table is a table which shows all the possibilities of truth and falsity.

Of a particular case, he writes

> The four rows of the truth table ... cover all possibilities.

And though Simpson doesn't say quite this in so many words, it isn't unfair to read him as taking it that *each* row of a table assigning values to some P, Q covers a possibility, a way the world could be with respect to the truth or falsity of these interpreted propositional letters.[2]

But of course, that's just wrong. We could, for example, have interpreted P to mean *Jack is taller than Jill* and Q to mean *Jill is at least as tall as Jack*. In which case the assignment of T to both P and Q is ruled out as impossible. That assignment of truth values is *not* one of the 'possibilities of truth and falsity', if that means worldly possibilities.

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Here's a much more sophisticated writer, Nick Smith in his *Logic*:

> Each row of the truth table represents a possible way of making propositions true or false. One of the rows (the actual row) represents the actual truth values of the propositions in question.

But he also writes

> A truth table lays out all possible combinations of truth and falsity of the basic propositions that make up a wff, and it shows the truth value of that wff in each of these cases.

[1] Originally published in 1988, this book is in now in its third edition and seems to have enjoyed some popularity. It has the merit of brevity; and I've heard that some students find it very clear.

[2] If he *didn't* mean this, he certainly should have said so!

Evidently in the first quoted passage, Nick Smith like Simpson is here thinking of the propositional letters recorded in a truth table as expressing propositions; thus understood, one row of assignments of values can represent the actual world. But in the second passage, the 'possible combinations' (judging from context) seem to be unconstrained by considerations of what they might represent – we are thinking of just any old matchings of letters to values.

So now consider e.g. Smith's following definition:

> A proposition . . . is satisfiable [just if] it is true in at least one row of its truth table.

What is the status of the conjunction $(\mathsf{P} \wedge \mathsf{Q})$? I think – from what Smith does and doesn't say hereabouts – that our response is supposed to 'satisfiable!'. Just take the first line of the table which lays out the assignment which matches both P and Q to the value T. Then their conjunction is T, showing that the proposition is T in at least one row of its truth table.

Remember again, however, that Smith wants to be working with interpreted letters. And we could be interpreting P to mean *Jack is taller than Jill* and Q to mean *Jill is at least as tall as Jack*. In which case the assignment of T to both P and Q is ruled out as impossible. And the conjunction $(\mathsf{P} \wedge \mathsf{Q})$ is then *not* satisfiable in the intuitive sense of being possibly true of the world. So what are we to make of Smith's definition?

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We'll be sorting out this sort of thing a bit later in *IFL*. For now, we just want the following point. We do need to distinguish two different senses in which lines on a truth table might be thought to represent possibilities:

(1) We could be dealing with mere *combinatorial* possibilities. Each line represents one way in which propositional letters (quite irrespective of their interpretation) may be matched up with 'T's and 'F's. In a standardly arranged truth-table, each 'mathematically possible' combination of atoms and values gets one and only one line. [Simpson in one place also talks of 'combinations', but makes nothing of it.]

(2) We could be dealing with *worldly* possibilities. A line of the table involves interpreted propositions and describes a a way the world could possibly be.

What we have just been noting is the important point that not all combinatorial possibilities of assignments of values to atoms reflect worldly possibilities.

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We needn't suppose that our authors are confused about this! My complaint is that they aren't as explicit as they should be.[3] And we need an explanation for why – when it comes to doing propositional logic – it really is ok to start ignoring questions of worldly possibility and just go purely combinatorial. Something needs to be said here. That's what *IFL* §12.4 is for.

[3]Apologies if, in revisiting Simpson and Smith, I've missed an explicit remark!